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*Examination of the Mathematical Language Used by Primary School Fourth Grade Students**

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ABSTRACT

The purpose of this study is to examine the mathematical language of fourth-grade students. A nested multiple case design was used in the study. The study participants are 150 fourth-grade students (86 girls and 64 boys) attending three different primary schools in the same school district in Altınordu district of Ordu in 2016–2017 school year. The data were collected using the Mathematical Language Use Inventory. Both qualitative and quantitative analyses were used to evaluate the data at hand. Based on the quantitative analysis, the students were divided into the following three groups: low, medium, and high-level students. These groups were used as the sub-themes in the analysis of the qualitative data. Descriptive analysis was used to evaluate the qualitative data. Evaluations and comparisons were made on the status of the low, medium, and high-level students in terms of the use of mathematical language. The findings of the evaluations and comparisons on the low, medium, and high-level students revealed that the use of mathematical language varied from student to student.

Keywords: Mathematical language, problem solving, problem posing

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Introduction

The main goal of mathematics teaching is not to transmit information to students but to equip the student with communication, association, reasoning, and problem-solving skills that will enable them to access information (National Council of Teacher of Mathematic [NCTM], 1989). Generally, while teaching mathematics in schools, teachers use a set of relations, rules, and symbols without basing mathematical knowledge on a reason. Therefore, students can only solve problems similar to the ones they have learned (Brown and Walter, 1983; Leikin et al., 2013; Olkun and Toluk, 2014). Individuals who learn mathematics as a set of rules fail to overcome the problems they encounter in real life because they cannot learn to think mathematically. Therefore, a focus on mathematical thinking, mathematical language, and communication should be the main target of teaching mathematics at school (Anghileri, 2005). Thinking and communication are interrelated processes. Explaining the use of mathematical language in this process can be accepted as a starting point in learning mathematics (Ferrari, 2004). Understanding the nuances of a child's mathematical language is necessary to understand the mathematical difficulties that the child experience (Warren, 2006).

In mathematics classes, students communicate mathematically by speaking, writing, and explaining their mathematical ideas. They learn to communicate through mathematical language by exploring their ideas in verbal activities in the classroom. There is no better way to express an idea or convey it to others (Van de Walle et al., 2014). Using mathematical language in discussions based on verbal expressions exchanged between among students and between a student and a teacher is a part of effective teaching. As a result of these discussions, a shared meaning is constructed about the discussed idea. In other words, the primary purpose of mathematics activities is to construct meaning (Olkun and Toluk, 2014). Mathematical language is used to construct and enhance meaning related to basic arithmetic content and define, compare and develop mathematical ideas (Warren, 2006). Individuals construct their knowledge by thinking effectively and deeply. They also construct understanding by using their existing ideas and knowledge (Van de Walle et al., 2014). To understand how students learn and understand and use their internal representations, structured math activities are a good starting point (Lesser and Tchoshanov, 2005).

The language of mathematics is a living language that enables concepts to be perceived, has a unique syntax, and gives birth to new concepts when necessary (Karaçay, 2011). Mathematical language is a part of daily life, and its meaning can change depending on the contex. For example, zero can mean absence, a bad result in an exam, or a number in a phone number. Teaching mathematical language is important as it increases students' motivation and moves students beyond the rules (Schoenfeld, 2016). Children learn how to use the mathematical language more effectively when conveying the results of their mathematical thoughts about the solution to a problem, verbally or in writing (Olkun and Toluk, 2014). Teaching mathematics as a language is often a subject that is given little attention and emphasis in the classroom. However, for students to become a true mathematical literate, it is necessary to bring real-life situations into the classroom and make them a part of the learning environment (Adams, 2003). The basic element of accomplishing mathematical thinking by acquiring concepts and information about mathematics is using mathematics field language correctly (Lansdell, 1999, as cited in Toptaş, 2015). With the correct use of the field language, students can understand abstract concepts more easily, reach new concepts and information in mathematics, and apply their mathematical knowledge and skills to different disciplines. These skills form the basic components of mathematics learning (Yeşildere, 2007).

Revealing and analyzing the mathematical language that children use allow creating insights about how children understand mathematics, how they learn it and the situations in which they have difficulty in learning (Pirie and Schwarzenberger, 1988). The way students represent mathematical concepts is affected by their beliefs, concepts, misconceptions, and individual differences (Duval, 2006). Students need to learn how to use mathematical language to communicate with each other and with their teachers. Transferring students' clear and consistent arguments and written and oral presentations that reflect their mathematical styles to the learning environment effectively develop students' mathematical language skills (Schoenfeld, 2016).

It is seen that most of the studies in mathematical language in Turkey examined secondary school students and only one study examined preschool students only (Dur, 2010; Çakmak, 2013; Taşkın, 2013; Yüzerler, 2013; Ünal, 2013; Akarsu, 2013; Kula-Yeşil, 2015; Yıldız, 2016). To the best of our knowledge, no study in mathematical language examined primary school students in Turkey. In this sense, this study aims to make important contributions to the literature. Mathematics is a way of thinking, which everyone uses to solve every-day problems (Yıkmış, 2012). It should be aimed that young children have a complete understanding of the mathematical concepts and principles forming the basis of mathematics as they learn from their teachers from the moment, they start their education life (Haylock and Cockburn, 2014). Therefore, this study aims to help teachers understand how students make sense of mathematical thoughts and information that form the basis of what they teach. Using an approach to support the development of problem-solving behaviors in basic education, especially in the first years, helps students both increase their success in problem solving, better understand the principles and subjects of mathematics, develop positive attitudes toward mathematics, and increase their self-confidence (Baykul, 2016, p. 83).

For this reason, it is important to examine the mathematical language that primary school students use while solving and posing problems. In this sense, the primary purpose of this study is to examine the mathematical language used by primary school fourth-grade students. Within this context, the sub-purposes of the study are as follows:

1. Examination of the mathematical language used by the fourth-grade students in the problem-solving process.
2. Examination of the mathematical language used by the fourth-grade students in the problem-posing process.
3. Examination of how the use of mathematical language in the problem-solving and posing processes of primary school fourth-grade students differs by the scores they get from the rubric.

It would be possible to say that the scope of this study is limited to the problem-solving and problem-posing processes, fourth- grade students, and the learning area of numbers.

Method

A nested multiple case study method, one of the qualitative research approaches, was used to examine the mathematical language used by students while solving and posing problems. This method is used to examine the mathematical language used by fourth-grade students while solving and posing problems and its characteristics. Also, it was aimed to identify the mathematical language they used within the context of in-class learning and experiences in the learning area of numbers. In this study, the nested multiple case study method was preferred as it would allow investigating how primary school fourth-grade students used mathematical language while solving and posing problems, whether the mathematical language used by them while solving and posing problems varied depending on the score they got from the rubric and the source of variation, if there was any. This method allowed making detailed implications about the mathematical language that is the study subject and examining more than one case associated with the mathematical language.

Participants

The study used a purposive sampling method because of the necessity of selecting cases that contain rich information for the depth of the study. In this regard, to determine the study participants, the data were collected through the interviews conducted with the school principals and primary teachers during the previous visits. Thus, three different schools in the same school district were selected. In these three schools, it was aimed to understand the use of mathematical language in the problem-solving and posing processes of primary school fourth-grade students within the context of in-class learning and experiences in the learning area of numbers. The reason for selecting fourth-grade students is that this grade level is the last grade of primary school. The study participants are 150 fourth-grade students (86 girls and 64 boys; their mean age is 9.5, SD=.57) attending three different primary schools in the same school district in the Altınordu district of Ordu. Initially, there occurred a concern that data saturation would not be reached in the study as activities such as thinking aloud and writing in detail could not be used in the mathematics lessons of primary school students, especially when using problem-solving skills and that the students had limited experiences. Thus it was decided to work with a large number of participants. As a result, a total of 190 students participated in the study. The papers of 40 students were excluded from the study as their papers were not readable or as they did not complete the tasks in the inventory. Instead of using the real names of the students, codes were assigned to each student such as Student 1 and Student 150. Before the application, the students were informed about the study, and their consent was obtained, and the students who did not want to participate in the study were excluded from the study.

Data Collection Tool and Rubric

The study data were collected using the “Mathematical Language Use Inventory.” A rubric was used to evaluate the data at hand.

Mathematical Language Use Inventory

The literature was examined to decide whether there is a need for an inventory to determine the use of mathematical language in the problem-solving and posing processes. As there was no measurement tool to determine the mathematical language used by students in the problem-solving and posing processes, an inventory was developed. The purpose of this inventory is to determine the mathematical language that students use when dealing with problems. In line with this purpose, first, the relevant literature was reviewed to establish the inventory's conceptual bases and determine the structural features it should have (Baykul, 2016; Gonzales, 1998; Mayer and Hegart, 1996; Polya, 2004). To reveal students' use of mathematical language while solving and posing problems, four operations skills, problem-solving and posing skills in the primary school fourth-grade curriculum were taken as a basis. While preparing a mathematical language use inventory covering four operations (addition-subtraction-multiplication-division), problems to measure the use of mathematical language while solving and posing problems within limits specified in the fourth-grade Mathematics Curriculum (Ministry of National Education [MoNE, 2015]); the problems used in the problem-solving achievement test developed by Özsoy and Kuruyer (2012), TIMSS (2011) questions and primary school mathematics fourth-grade teacher's guide book (Karakuyu & Şenyurt, 2016) were drawn on. After the inventory was created, the draft was sent to three experts having a doctorate in Primary Education and having experience in mathematics teaching and one expert having a doctorate in Measurement and Evaluation in Education to obtain their opinion about the draft. The inventory was finalized the experts' feedbacks provided. The Lawshe method was used for the content validity index (CVI).

The content validity index was calculated as .92. A pilot study was conducted to determine whether the tasks in the inventory reveal the mathematical language in the problem-solving and problem-posing processes and evaluate the data collection process and the problems in the inventory. There were 10 problems and eight subtasks for each problem prepared in accordance with the problem-solving stages of Polya (2004) in the inventory prepared for the pilot study. These tasks are as follows; (a) Explain what is given and asked for in the problem, (b) Which mathematical operation(s) will you use to solve the problem?, (c) Explain why you chose the mathematical operation(s) to use, (d) Solve the problem, (e) Can you solve the problem by drawing a picture or figure? If so, how? (f) Write a problem similar to the one you have solved, (h) What is/are the mathematical operation(s) that should be used to solve the problem you have written? (i) What is the clue to solve the problem you have written? Please explain briefly.

The pilot study was conducted with 50 students who were excluded from the study group in two course hours. As a result of the pilot study, three problems in the inventory were removed and the inventory was finalized, making it ready for the main application. In the final form of the inventory, there were seven problems in total and subtasks to determine the use of mathematical language for these problems. In addition, the mathematical language skills that the problems in the inventory aim to reveal are as follows: (1) being able to translate verbal language into symbolic and visual language (problems 1, 2, 3, 4), (2) being able to translate visual language into a verbal and symbolic language (problems 5, 6), (3) being able to translate symbolic language into a verbal language (problem 7).

The Rubric

To determine the mathematical language used by the students during problem solving and posing processes, a rubric were developed by the researchers to evaluate the tasks presented to the students through the inventory. To develop the rubric, first, the literature was reviewed (Altun, 2005; Altun, 2015; Baykul, 2014; Çalıkoğlu-Bali, 2002; Olkun and Toluk, 2014; Van de Walle et al., 2014). While developing the rubric, the skills required to use of the mathematical language suitable for the current study were taken into account. These skills can be listed as follows: translating verbal language into symbolic language, translating verbal language into visual language, translating symbolic language into verbal language, translating symbolic language into visual language, translating visual language into verbal language, and translating visual language into symbolic language. In the measurement of the performance indicators in the rubric, of the following grading levels were used: expressing correctly and thoroughly = 2 points, expressing incompletely = 1 point, and expressing wrongly or expressing nothing = 0 point. After developing the rubric, the opinions of six experts having a doctorate in Primary Education were sought to ensure content validity. The Lawshe method was used for the content validity index. The CVI value was found to be .94. The criteria of the rubric aim to determine the level at which students use the mathematical language. The rubric is composed of seven performance indicators about problem-solving or posing processes. For each performance indicator, the students using mathematical language thoroughly and correctly were given 2 points, using mathematical language incompletely or insufficiently were given 1 point, while 0 point was given to whom could not use mathematical language and the scores obtained from the Mathematical Language Use Inventory were summed. The total mathematical language use score of each student was obtained. For the reliability of the rubric, all the data obtained from the Mathematical Language Use Inventory were evaluated by the researchers and three other field experts, and the correlation coefficient between the results of the five researchers was calculated to be .98. The lowest score to be taken from the mathematical language use inventory is 0, while the highest is 74.

Application of the Data Collection Tool

Before starting the application process, permissions were obtained from the Ordu Provincial Directorate of National Education and the principals of the schools where the application would be made were contacted to inform the study. Then, the fourth-grade teachers in whose classes the data would be collected were contacted to determine the most convenient time for the application of the Mathematical Language Use Inventory to their students. The inventory was then administered to the students during the course hours determined by their teachers. While talking to the teachers before the application, information was obtained about the mathematics knowledge level of the students and the way they use to teach mathematics lessons verbally. Before the application, the primary researcher who would conduct the application spent some time with the participants for them not to be negatively affected by the presence of the researcher during the application. Data collection process was conducted in the spring term of the 2016–2017 school year. Before starting the main application, a pilot study was conducted and a sample application was performed by the primary researcher. The main application was carried out with the participation of fourth-grade students within 24 course hours.

Data Analysis

In the first stage of the study, it is aimed to evaluate the data quantitatively and qualitatively. The quantitative data obtained by evaluating mathematical language use inventories completed by the students through the rubric were transferred to the IBM SPSS 22 program. Kolmogorov- Smirnov test was used to determine whether the total scores of the participants showed a normal distribution. As the total scores obtained from the Mathematical Language Use Inventory were normally distributed (N= 150, *p*= .200), the range of 0 - (mean-standard deviation) was defined as the low (or subgroup), the range of (mean-standard deviation) - (mean+standard deviation) was defined as the medium and the range of (mean+standard deviation) < and the subsequent was defined as the high group. Thus, the range of (0-15.81) was specified as the low group, the range of (15.81-36.47) was specified as the medium group, and the range of 36.47< … was specified as the high group. The students were grouped as low-level students (N= 25), medium-level students (103), and high-level students (N= 22) according to the scores they obtained from the Mathematical Language Use Inventory. These groupings were determined as sub-themes. The qualitative data obtained from the Mathematical Language Use Inventory were evaluated through a descriptive analysis method. The data obtained through this method were summarized and interpreted according to the previously determined themes. The main purpose of the descriptive analysis is to present the findings to the reader in an organized and interpreted manner (Yıldırım and Şimşek, 2008, p. 224). In the analysis of the data, the following stages were followed: (a) determining the themes and sub-themes (b) transcribing the data (c) organizing the data according to the themes (e) analyzing and interpreting the data. The themes and sub-themes determined within the context of the study are presented in Table 1.

Table 1. Themes and sub-themes

	Themes	Sub-themes
Themes	Being able to use verbal language in the process of understanding the problem	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to use verbal language in the process of determining the operation	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to use verbal language while performing the operations selected	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to use symbolic language in the process of solving the problem	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to use visual language in the process of solving the problem	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to translate symbolic and/or visual language into a verbal language in the process of posing the problem	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students
	Being able to use verbal language in the process of comprehending the problem	The status of the low-level students
		The status of the medium-level students
		The status of the high-level students

In Mathematical Language Use Inventory, the sub-theme of being able to use verbal language in understanding the problem was prepared to explain students' skill of using verbal language in the process of understanding the problem. The student, who can use verbal language to understand the problem, can clearly express what is given and asked for in the problem and can explain the problem in his/her own words. To explain how the evaluation is made according to Sub-Theme 1, the analysis of the answers given by Student 135 (low-level student), Student 124 (medium-level student) and Student 132 (high-level student) to the question *“What is given and asked for” in the problem 1 are given as examples. The answer given by Student 135 is “A mountaineer climbs 1315 meters, then takes a break and climbs back up. How many meters are left for the mountaineer to reach the summit?”; the answer given by Student 124 is “He climbs 1315 meters, climbs 915 meters more, the remaining distance is 530 meters. How many meters is the mountain?” and the answer given by Student 132 is “A mountaineer takes a break after climbing 1315 meters and climbs another 915 meters after the break and the remaining distance is 530 meters. The height of the mountain.’’* In the answers, it is seen that Student 135 expressed the given information incompletely. Student 124 could not explain what was given in his/her own words. Student 135 and Student 124 could not use verbal language adequately in the process of understanding the problem. The performance of Student 132 was interpreted as follows: “He/she was able to use verbal language in the process of understanding the problem by expressing clearly what was given and what was asked for.” The data obtained from the Mathematical Language Use Inventory allowed obtaining in-depth information for the determination of the mathematical language use of the primary school fourth-grade students while solving and posing problems. The researchers and three independent experts examined the themes, sub-themes, and findings identified during the data analysis process. The internal validity of the study was attempted to be ensured by reporting the obtained data in detail, direct quotations, and explaining the findings objectively based on these quotations. The external validity of the study was attempted to be ensured by explaining the research process and what was performed in this process in detail. To establish the reliability of the study, all data were reviewed by three field experts. Using the reliability formula proposed by Miles and Huberman (1994), the *p-value* was found as .94.

Findings

Being Able to Use Verbal Language in the Process of Understanding the Problem – the Status of the Low, Medium, and High-Level Students

The findings of the study revealed that the low-level students had difficulty expressing the problem in their own words. It was observed that they confused concepts such as height and size (Students 62, 63), only expressed what was given or only what was asked for (Students 84, 87, 95,150), and incompletely expressed what was given (Students 60, 68, 96). According to the findings, it was observed that the students who did not understand the problem could not clearly express what was given and what was asked for using verbal language. The mistakes made by the medium-level students at this stage include using numerical data only (Students 35, 106, 144) and incomplete or incorrect data (Students 32, 112, 122) while expressing what was given and writing the same problem sentence while expressing what was given and what was asked for (Students 69, 75). The medium-level students were observed to express what was given and asked for by summarizing and using verbal language to understand the problem (Students 77, 98, 123). The high-level students were able to articulate clearly what was given and asked for in the problem (Students 2, 15, 18, 19, 43, 81, 82, 129). Findings revealed that they could successfully use verbal language in the process of understanding the problem. Examples illustrating this situation are presented below.

Size of the mountain. (Low-level – Student 63, Problem 1)

A mountaineer climbs 1315 meters, climbs 915 meters after the break, the remaining distance is 530 meters to reach the summit. What is the height of the mountain? (Middle-level – Student 75, Problem 1)

The mountaineer climbs 1315 meters, takes a break, then climbs 915 meters. The remaining distance is 530 meters to reach the summit. Find how many meters the mountain is. (High-level – Student 82, Problem 1)

Being Able to Use Verbal Language in the Process of Determining the Operation – the Status of the Low, Medium, and High-Level Students

In the process of determining the operation, the low-level students were observed to commit mistakes, such as selecting the wrong operation for the solution of the problem (Students 21, 62, 95, 102), selecting unnecessary operations in the solution to the problem (Students 59, 83), using incomplete operations for the solution of the problem (Students 101, 135), confusing mathematical operations with the numerical values in the problem (Student 14). Based on the mistakes made by these students, it was concluded that they were not successful in using verbal language in the process of determining the operation. The medium-level students who could express what was given and what was asked for in the problem also correctly expressed the operations to solve the problem (Students 96, 144, 146). Some students who determined the correct operations for the problem's solution wrote by confusing the order of the operations (Students 21, 61). The students who could not express what was given and asked for in the problem failed to determine the operation (Students 3, 76). Some students wrote multiplication or division operations without focusing on the main idea in the problem sentence, as the term "times" was used in the problem. Some students chose random operations among the numerical data given in the problem (Student 83, 145). Some medium-level students identified the correct operation for a part of the problem (Students 38, 110). The high-level students expressed the operations to be followed in solving problems in order and complete. They were successful in using verbal language in the process of determining the operation. They successfully used verbal language in determining the operation (Students 22, 50, 64, 73, 125, 118, 128, 132). Some examples illustrating such situations are presented below:

Division, multiplication (Low-level – Student 101, Problem 2),

Subtraction, division, multiplication (Medium-level – Student 61, Problem 2)

Division, multiplication, subtraction (High-level – Student 128, Problem 2)

The correct order in the solution to Problem 2[1] is as follows: Division, multiplication, subtraction.

Being Able to Use Verbal Language While Performing the Operation Selected – the Status of the Low, Medium, and High-Level Students

While justifying the operation, the low-level students wrote down what was asked for in the problem (Students 21, 60, 100) or used expressions such as *the result was found like that* (Students 63, 100). There were also students expressing which operations to apply (Student 60, 65). They could not accurately justify every operation they used in the solution (Students 66, 90, 105). It was determined that the low-level students could not use verbal language correctly while justifying the operations they selected. It was observed that the low-level students could not express the main idea that would lead to a solution to the problem, or they expressed it incorrectly.

[1] Problem 2. A tanker is completely filled with oil and there are 105 liters of oil in this tanker. How many liters of oil are left when 5/2 of this oil is used?

While justifying the operations chosen by the medium-level students, they were observed to frequently use expressions such as "*the result was found like that*," "*to solve the problem*" (Students 11, 13). Again, several students reported that they chose those operations to find what was asked for. Still, it was seen that they did not specify why they would apply the operations they chose to which numerical data are given in the problem (Students 16, 36, 40, 139). No medium-level students who explained why they chose which operations step by step were found. The high-level students stated that if there was the term "times" in the problem, they would multiply, if they were asked for the remainder, they would subtract, and when the total was asked for, they would add. It was observed that they used expressions such as "I would divide and multiply" while performing operations including fractions, but they could not explain why they used division and multiplication operations (Students 1, 107, 117, 129). They partially used verbal language while justifying the operations they had chosen (Students 22, 132). Some students incompletely expressed the main idea that would lead to a solution to the problem. Some examples illustrating such situations are presented below:

> *In order to solve the problem* (Low-level – Student 63, Problem 3)
>
> *As it was asked how much money he/she spent* (Medium-level – Student 139, Problem 3)
>
> *Since it says 6 meters, 19 TL for each meter, we multiply it and add the result with 280 TL because it says how much it costs* (High level – Student 22, Problem 3)

Being Able to Use Symbolic Language in the Process of Solving the Problem – The Status of the Low, Medium, and High-Level Students

As the low-level students were unable to determine the correct option for the solution, they were observed to experience difficulties in using the symbolic language of mathematics in the process of solving the problem (Students 68, 83, 99). It was observed that these students tried to solve the problems randomly using the numerical data given in the problem and the four operations (Students 65, 86, 87, 95). It was observed that they could not reach the correct result. The students who could not use the symbolic language correctly in solving the problem were observed to be unsuccessful in summarizing the problem with their own words, determining the operation for the solution of the problem, and justifying the operation they chose. The findings reveal that the medium-level students could not reach the correct result because they made operational errors while using a symbolic language to solve the problem (Students 88, 120). While solving the problem, it was observed that they left the task uncompleted without reaching what was asked for (Students 27, 98). It was seen that they performed wrong operations and correct operations in solving the problem (Students 10, 116). Although students (Students 23, 29, 55) were unable to express the main idea that would lead to a solution to the problem and were engaged in the wrong solution, it was observed that the medium-level students could also use symbolic language while solving problems (Students 108, 121). According to the findings, the high-level students determined a complete plan for the solution and implemented the plan they determined without making any mistakes. They successfully used symbolic language to solve the problem (Students 15, 18, 43, 64, 82, 125, 129). Some examples illustrating such situations are presented below:

> *264×5 =1320 (Low-level – Student 47, Problem 4)*
>
> *264:6=44 44×2=88 264-88=176 (Medium-level – Student 10, Problem 4)*
>
> *k+5k=6k, 264:6=44, 44×5=220 large number (High-level – Student 125, Problem 4)*

The correct solution to Problem 4[2]: 1 time + 5 times = 6 times (expression of the sum of the smaller number and the larger number), 264:6=44 (smaller number), 44×5=220 (larger number), or 1 time + 5 times = 6 times (expression of the sum of the smaller number and the larger number), 264:6=44 (smaller number), 264-44=220 (larger number).

Being Able to Use Visual Language in the Process of Solving the Problem – The Status of the Low, Medium and High-Level Students

The low-level students could not draw a shape or diagram suitable for events and relationships in solving the problem (Students 14, 21, 47, 60, 87, 101, 149, 150). These students were observed to draw pictures evoking the concepts in the problem but that were not solution oriented. Therefore, it would be safe to say that they were unable to use the visual language of mathematics to solve the problem. Among the middle-level students, the students who drew the figure or diagram incompletely or incorrectly or made the necessary markings incompletely or incorrectly constituted the majority (Students 25, 28). Some pictures and drawings do not fit the visual language of mathematics and are not solution oriented (Students 5, 6, 136). Students could not show the data appropriately in the figures they drew (Students 73, 113). Thus, it can be said that these students could not comprehend the visual language of mathematics because they used symbols instead of shapes or diagrams (Students 6, 25, 17, 28, 71, 131, 136, 140). The high-level students were mostly able to solve the problem by drawing shapes and diagrams suitable for the events and relationships in the problem. It was observed that the high-level students could not express the sizes of the figures showing the same unit equally in their drawings (Students 12, 132). In addition, there are deficiencies in the markings on the figures or diagrams suitable for the events and relations in the problem (Students 3, 22). It was observed that the high-level students were relatively more successful in using visual language to solve the problem than the low and medium-level students (Students 39, 82, 117, 125, 133). Examples illustrating these situations are presented below (Figure 1).

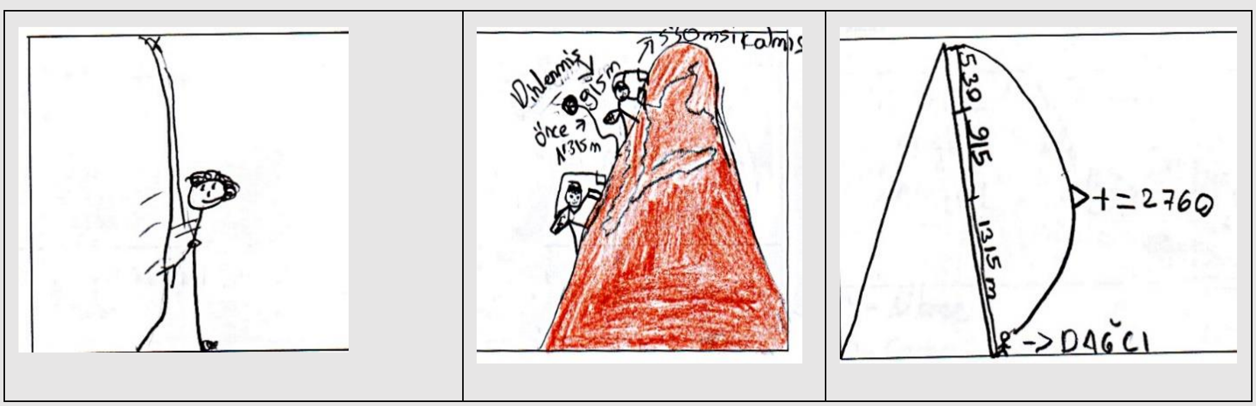

Figure 1. Sample Drawings for Problem1[3]: Drawing 1. Low-Level – Student 150, Drawing 2. Medium-Level – Student 136, Drawing 3. High-Level -Student 39

[2] Problem 4. The sum of two different numbers is 264. If the larger number is five times the smaller number, what is the larger number?

[3] Problem 1. A mountaineer takes a break after climbing 1315 m. After the break, he climbs another 915 m. If the mountaineer has 530 meters left to reach the summit, what is the height of the mountain?

Being Able to Translate Symbolic and/or Visual Language into Verbal Language in the Process of Posing the Problem and Being Able to Pose a Problem Similar to the Problem Already Solved – The Status of the Low, Medium and High-Level Students

As the low-level students were unable to interpret the visual shape correctly, the problems they posed according to the shape given in the task (Students 47, 60, 86, 87, 95, 101, 135, 149) were wrong. Therefore, the problems they posed were either incomplete or incorrect in terms of meaning and numerical data. It was observed that the low-level students could not pose an original problem. Some of the medium-level students did not change the data and conditions of the problem but posed the problem changing its subject (Students 12, 20) only, some of them posed a similar problem by changing the data and the subject without changing the conditions (Students 7, 25, 88), and some of them posed a similar problem simply by changing the data (Students 33, 124). Only one student was able to pose a problem by changing the data, subject, and conditions (Student 140). In the problems that the high-level students posed, it was observed that they posed similar verbal problems to the problems they had already solved (Students 1, 2, 15, 22). Some examples illustrating such situations are presented below:

An apple weighs 50 grams, when 2 more apples are added, they weigh 150 grams, how much would it weigh, if you put 6 more apples next to these apples? (Low-level – Student 87, Problem 5)

I have 4 friends with me, and I have 36 hazelnuts, how many hazelnuts does each of my friend will have? (Student 86, Problem 1)

A scale weighs 3 apples as 220 grams. How many grams is an apple? (Medium-level – Student 31, Problem 5)

What is the most preferred color? (Medium-level – Student 17, Problem 6)

Three apples weigh 220 grams. The weight of the first apple is 75 grams, the weight of the second apple is 25 grams more than the first apple. How many grams is the third apple? (High-level – Student 125, Problem 5)

Favorite colors in a classroom are red, green, blue, and yellow. As 4 students are choosing the color red, 2 students choosing the color green, 6 students choosing the color blue and 7 students choosing the color yellow, so how many students are there in the classroom? (High-level – Student 126, Problem 6)

Being Able to Use Verbal Language in the Process of Comprehending the Problem – The Status of the Low, Medium and High-Level Students

The low-level students had difficulties posing problems by transforming the visual, verbal, and symbolic language of mathematics to each other. They wrote problem sentences, including incomplete, incorrect, contradictory, and inadequate statements in terms of meaning. Therefore, they could not express the main idea to be used in solving the problems they wrote (Students 65, 95, 101). Although the medium-level students themselves posed the problems, they could not accurately and thoroughly express the main idea that they expressed in the problem and used to solve the problem (Students 71, 127, 130, 137, 142). Students expressed what was given or asked for as the main idea to be used in the solution. Some students expressed the main idea to solve the problem incompletely or incorrectly (Students 54, 131). Therefore, medium-level students were inadequate in using verbal language to comprehend the problem. However, although there were students (Students 12, 127) who could not interpret visual shapes and tables correctly, it was observed that there were students who were successful in translating visual language into a verbal language (Students 71, 109). It was observed that some students expressed the main idea to be used in the solutions to the problems posed by the high-level students correctly, while some students expressed it incompletely (Students 71, 128). This shows that the high-level students were able to partially use verbal language to comprehend the problem (Students 22, 81). These students were successful in posing problems by translating visual language into verbal language and symbolic language into a verbal language (Students 39, 43, 73, 82, 107, 117, 118, 125, 126, 132). Some examples illustrating such situations are presented below:

> *We shared 336 bagels equally among 28 people. 8 people each ate 1 bagel of one of these 28 people, so how many bagels of this person were left?*- What is the clue to be used in the solution to the problem you have written? Explain it briefly. *People (Low-level – Student 65)*
>
> *He/she formed the following question: 336 feeds will be equally divided among 28 cows. What is left if we subtract 8 from this?* What is the clue to be used in the solution to the problem you have written? Explain it briefly. *Divide and subtract (Medium-level – Student 78)*
>
> *What is 8 minus one 28 of 336 marbles?* What is the clue to be used in the solution to the problem you have written? *One 28 of 336 means division and minus 8 means subtraction (High-level – Student 22).*

Results and Discussion

In mathematics teaching, it is possible to say that the approach involving the direct presentation of the information by the teacher and then students' working on four operations skills and similar skills are not effective in students' learning and keeping mathematical concepts in mind (Van de Walle et al., 2014). Observing the student in the problem-solving process allows us to recognize and understand the student's thinking structure (Polya, 2004). In the problem-solving process, emphasis should be placed on the solution rather than the answer to the problem. It should be emphasized how the student solves the problem, what information in the problem contributes to this solution, how he/she represents the problem (table, figure, concrete object, etc.), how the chosen strategy and the way of representation facilitate the solution. Ways of solving a problem should not be given to students directly; rather a suitable environment should be provided for students to create their own solutions. In addition, students should be given the opportunity to pose unique problems similar to and different from the problems they have solved before. Primary school level has critical importance in teaching and developing problem-solving skills. However, it can be argued that this does not make students internalize mathematical skills because in primary school, first, basic concepts of mathematics, operation skills, and problem-solving skills that require operation skills are taught. However, more emphasis is put on mathematical operations skills rather than teaching children mathematical thinking skills by using the language of mathematics in schools.

Thus, teachers and students are more interested in the results of operations rather than constructing meaning, reasoning, and problem solving by thinking in a mathematical language. In this sense, the mathematical language used by primary school fourth-grade students in solving and posing problems was examined in the current study. Based on the findings, it was observed that the low-level students were not be successful in using verbal language by not correctly expressing what was given and what was asked for in the process of understanding the problem. It was observed that the medium-level students were mostly successful in using verbal language by correctly summarizing what was given and asked for in understanding the problem. The high-level students were able to use verbal language successfully to understand the problem by clearly expressing what was given in the problem and what was asked for. To solve a problem, a connection should be established between what is given and what is asked for by using the concepts of mathematics, reasoning, and operations. Students who cannot establish this connection are unable to successful in problem solving (Baykul, 2016; English et al., 2008; Polya, 2004). The results of the study are also in line with the findings of the previous studies. It was observed that students who were unable to express what was given and asked for in the problem using verbal language were successful in solving the problem, while students who were unable to summarize the problem in their own sentences using verbal language were unsuccessful at this stage.

It was observed that the low-level students were not successful in using verbal language to determine the operation. Although middle-level students confused the order of the operations to be followed in solving the problem or expressed them incompletely, it was observed that the majority of the middle-level students were successful in using verbal language in the process of determining the operation. The high-level students were observed to be successful in using verbal language by expressing the operations to be used in the solution to the problem in order and completely during determining the operation. At the stage of understanding the problem, what is given and what is asked for in the problem is clearly determined, and an answer is sought for the relations between what is given and what is asked for (Altun, 2015; Gonzales, 1998). Students who succeed at this stage can express the operations to be followed in solving the problem in order and complete. The results obtained in this study confirm this conviction for high-level students.

It was observed that the low-level students could not express the main idea that would lead to the problem's solution, or they expressed it incorrect. No student explained step by step why he/she chose which operation among the middle-level students. It was observed that there were students who were unable to express the main idea that would lead to the solution of the problem or expressed it incomplete or incorrect. It was observed that the high-level students could partially benefit from verbal language while justifying the operations they chose and that some students incompletely expressed the main idea that would lead to the solution of the problem. Problems used to develop students' problem-solving skills in mathematics education are mostly in verbal form. For students to solve these verbal problems, they should understand the text of the problem and the numerical relations described in the problem and are able to establish relationships between them.

For this reason, verbal problems allow interactions between reasoning, mathematical development, and mathematical language formation (Aydoğdu and Olkun, 2004; Mason, 2003; Zazkis, 2000). The findings of the study are in line with the findings of previous studies. It was observed that students who were able to express the main idea verbally that would lead to a solution in the problem-solving process could use mathematical language more effectively. Knowing at what stage students make mistakes in the problem-solving process helps us understand where students have difficulties. In terms of mathematics education, it is important to define and evaluate students' skills in the problem-solving process (Baki et al., 2002).

The low-level students were mostly not successful in using the symbolic language of mathematics in the problem-solving process because they were unable to determine the right plan for the solution. It was observed that the medium-level students were mostly successful in using symbolic language in the problem-solving process. However, it was seen that while using symbolic language in the problem-solving process, they made operational mistakes and left the problem solving incomplete. The findings of the study revealed that the high-level students determined a complete plan for the solution and implemented the plan they determined without making mistakes. They were successful in using symbolic language in the problem-solving process. By controlling symbolic operations, it should be evaluated whether the students who make mistakes make them due to lack of procedural knowledge or lack of conceptual knowledge. Every answer given by students in problem solving and posing processes should be checked, regardless of whether their answers were correct or not, and students should be talked about their thinking processes. The correct and appropriate use of symbolic language is important in the acquisition of problem-solving skills. All these arguments are supported by the findings of this study because students who are unable to fully understand the expressions and concepts given in the problem and cannot establish the equations related to the problem have difficulty in solving problems (Mayer, 1982; Mayer and Hegart, 1996; Polya, 2004).

It was observed that the low-level students generally produced pictures or drawings that were not solution oriented. Therefore, it would be safe to say that they were not successful using the visual language of mathematics in the problem-solving process. Among the middle-level students, the students who drew the figure or diagram incompletely or incorrectly or made the necessary markings incompletely or incorrectly constituted the majority of the students. The high-level students were observed to be successful in using visual language in the problem-solving process. Drawing a suitable figure or diagram for a problem is a high-level indicator of the understanding of that problem (Altun, 2015; Baykul, 2016). This is supported by the results obtained in this study because the students who comprehended the problem were more successful at this stage.

It was observed that the low-level students were unable to pose original problems similar to the problem they solved. It was observed that while the low-level students posed problems by translating visual and symbolic language into verbal language, they posed problems that were problematic or erroneous in terms of meaning. The middle-level students had successful examples of posing problems similar to the problem they solved and posing problems by translating visual and symbolic language into verbal language. The high-level students were highly successful in posing problems similar to the problem they had solved and in posing problems by translating visual and symbolic language into verbal language. High-level students were good at translating verbal, visual, and symbolic forms of mathematical language into each other.

It was observed that the low-level students were unable to express the main idea to solve the problems they posed. It is possible to say that the medium-level students were inadequate in using verbal language to comprehend the problem. It was observed that the high-level students partially verbally expressed the main idea to solve the problems they posed. To solve a problem, it is not enough that students chose the right operations. What is needed to find a solution to a problem is to comprehend and express the main idea. This is possible using the verbal language of mathematics (Schleppegrell, 2007; Schoenfeld, 2016; Schunk, 2011). These findings support the literature because students who comprehend the main idea of the problem use verbal language more successfully.

The Mathematical Language Use Inventory and rubric can be used by teachers to focus on students' experiences of using mathematical language, identify the difficulties students experience in using mathematical language, and prepare learning environments to overcome these difficulties. In addition, it is thought that examining the use of mathematical language with larger study groups of different characteristics at the primary school level will be important in terms of revealing their current state. In this study, the mathematical language used by the fourth-grade students (mean age 9.5, SD= .57) to solve and pose problems was examined. It should be noted that the scope of this study is limited to the problem-solving and problem-posing processes and to fourth-grade students. Thus, further studies might examine different mathematical processes and the use of mathematical language at different grade levels (different age groups). Also, this study is limited to the learning area of numbers. It would be important to focus on different learning areas in future studies. Mathematical Language Use Inventory was used as the data collection tool of the study. The data were evaluated with a rubric developed by the researchers. To examine the use of mathematical language, further studies may focus on methods such as observation and interview. Further studies can also examine the opinions, and experiences of teachers and students. In addition, studies that will be designed quantitatively and qualitatively or in which these two designs will be used together can also be conducted to determine the mathematical language use of primary school students.

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